III. PARALLEL PATHWAY MODEL OF ANKLE DYNAMICS

Our laboratory, the Neuromuscular Control Laboratory, has developed a parallel pathway model (Fig. 1) to describe ankle dynamics [7]. This model treats the relationship between ankle angle and net ankle torque as the sum of a linear and nonlinear contributions. The upper, linear pathway models intrinsic stiffness as a second-order system with parameters corresponding to inertia (I), viscosity (B) and elasticity (K). The lower, nonlinear pathway models reflex stiffness as a cascade of a derivative, a time delay, a static nonlinearity (i.e., half-wave rectifier) defined as

(2)

and a low-pass system representing muscle activation. The latteris simplified to second-order, though in many cases it has beenshown to be better represented by a third-order filter [8], [9]. Asecond-order model is justified for the muscle activation sincewe assume that the “system” is a normal subject under passiveconditions [8], [10]. The parameters associated with thelow-pass system are damping parameter (ζ), natural frequency (ω)and gain (g).

*A. Discrete-Domain Approximation to a Derivative Versus Bilinear Transform*

The discrete-domain approximation to a derivative (Newton’s backward formula) maps points from the left-half plane into a circle of radius 1/2, centered at z=1/2 in the plane z[11], [12]. Since this mapping confines the discrete-time poles to low frequencies, its use is restricted to systems with low resonant frequencies [12]. This is a good approximation to a derivative provided the bandlimit of interest is confined to low frequencies. For our work the bandlimit of interest is 0.15 of the sampling rate (see Section V); therefore, this approximation is appropriate for approximating the intrinsic component of ankle dynamics. Conversely, the bilinear transform maps the left-half plane into the entire unit circle and, hence, does not have the same restrictions as above. The bilinear transform gives a better fit to the transient portion of a step response than does the discrete-domain approximation [11], [12]. For this reason, it is used to transform muscle activation dynamics, modeled as an infinitie impulse response (IIR) system, in Fig. 1.

However, in the finite impulse response case (intrinsic stiffness pathway Fig. 1) the bilinear transform cannot be used because, in general, it cannot transform an all-zero system into a stable discrete equivalent. (The bilinear transform is valid up to half the sampling rate, i.e., the Nyquist frequency.) Using the bilinear transform, a derivative operator in continuous-time transforms into a pole-zero system in discrete-time with a pole at z = -1. This discrete pole maps back to an unstable pole on the axis in the plane. For this reason, the derivative operator is transformed to discrete-time using Newton’s backward formula [13].